OCR
Oxford Cambridge and RSA

Tuesday 16 June 2015 – Afternoon

A2 GCE MATHEMATICS (MEI)

4769/01 Statistics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4769/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

* 4 4 2 7 7 2 3 6 0 4 *

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

Option 1: Estimation

1 The random variable X has the following probability density function, in which a is a (positive) parameter.

$$f(x) = \frac{2}{a} x e^{-x^2/a}, \quad x \geqslant 0.$$

(i) Verify that $\int_0^{\infty} f(x)\,dx = 1$. **[1]**

(ii) Show that $E(X^2) = a$ and $E(X^4) = 2a^2$. **[7]**

The parameter a is to be estimated by maximum likelihood based on an independent random sample from the distribution, $X_1, X_2, \ldots, X_n$.

(iii) Show that the logarithm of the likelihood function is

$$n \ln 2 - n \ln a + \sum_{i=1}^{n} \ln X_i - \frac{1}{a} \sum_{i=1}^{n} X_i^2.$$

Hence obtain the maximum likelihood estimator, $\hat{a}$, for a.

[You are not required to verify that any turning point you find is a maximum.] **[7]**

(iv) Using the results from part **(ii)**, show that $\hat{a}$ is unbiased for a and find the variance of $\hat{a}$. **[5]**

(v) In a particular random sample from this distribution, $n = 100$ and $\sum x_i^2 = 147.1$. Obtain an approximate 95% confidence interval for a. (You may assume that the Central Limit Theorem holds in this case.) **[4]**

Option 2: Generating Functions

2 The random variable Z has the standard Normal distribution. The random variable Y is defined by $Y = Z^2$.

You are given that Y has the following probability density function.

$$f(y) = \frac{1}{\sqrt{2\pi y}} e^{-\frac{1}{2}y}, \quad y > 0.$$

(i) Show that the moment generating function (mgf) of Y is given by

$$M_Y(\theta) = (1 - 2\theta)^{-\frac{1}{2}}.$$ **[6]**

(ii) Use the mgf to obtain $E(Y)$ and $\text{Var}(Y)$. **[5]**

The random variable U is defined by

$$U = Z_1^2 + Z_2^2 + \ldots + Z_n^2,$$

where $Z_1, Z_2, \ldots, Z_n$ are independent standard Normal random variables.

(iii) State an appropriate general theorem for mgfs and hence write down the mgf of U. State the values of $E(U)$ and $\text{Var}(U)$. **[4]**

The random variable W is defined by

$$W = \frac{U - n}{\sqrt{2n}}.$$

(iv) Show that the logarithm of the mgf of W is

$$-\sqrt{\frac{n}{2}}\theta - \frac{n}{2}\ln\left(1 - \sqrt{\frac{2}{n}}\theta\right).$$

Use the series expansion of $\ln(1 - t)$ to show that, as $n \to \infty$, this expression tends to $\frac{1}{2}\theta^2$.

State what this implies about the distribution of W for large n. **[9]**

Option 3: Inference

3 At an agricultural research station, trials are being carried out to compare a standard variety of tomato with one that has been genetically modified (GM). The trials are concerned with the mean weight of the tomatoes and also with the aesthetic appearance of the tomatoes.

(a) **(i)** Tomatoes of the standard and GM varieties are grown under similar conditions. The tomatoes are weighed and the data are summarised as follows.

Variety	Sample size	Sum of weights (g)	Sum of squares of weights (g^2)
Standard	30	3218.3	349 257
GM	26	2954.1	338 691

Carry out a test, using the Normal distribution, to investigate whether there is evidence, at the 5% level of significance, that the two varieties of tomato differ in mean weight.

State one assumption required for this test to be valid. **[10]**

(ii) The data in part **(i)** could have been used to carry out a test for the equality of means based on the t distribution. State **two** additional assumptions required for this test to be valid.

Discuss briefly which test would be preferable in this case. **[4]**

(b) In order to judge whether, on the whole, GM tomatoes have a better aesthetic appearance than standard tomatoes, a trial is carried out as follows. 10 of each variety are chosen and a consumer panel is asked to arrange the 20 tomatoes in order according to their appearance.

(i) State **two** important features of the way in which this trial should be designed.

Comment briefly on how reliable the evidence from the trial is likely to be. **[3]**

(ii) The order in which the consumer panel arranges the tomatoes is as follows. The tomato with best appearance is listed first. G and S denote GM and standard tomatoes respectively.

$$G \quad G \quad G \quad S \quad G \quad G \quad G \quad S \quad G \quad S \quad S \quad S \quad G \quad G \quad S \quad G \quad S \quad S \quad S \quad S$$

Carry out an appropriate test at the 1% level of significance. **[7]**

Option 4: Design and Analysis of Experiments

4 **(a)** The standard one-way Analysis of Variance (ANOVA) model is expressed in the usual notation as follows.

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$$

(i) Explain what the terms Y_{ij}, μ, α_i and ε_{ij} represent. **[4]**

(ii) State a distributional assumption about ε_{ij} and explain briefly why this assumption is required. **[4]**

(iii) State the null and alternative hypotheses for the usual one-way ANOVA test. Explain clearly how to interpret the two possible outcomes of the ANOVA test. **[4]**

(b) I drive frequently between two cities, A and B. There are k different routes that I can take. On each of these routes the journey time varies according to time of day, traffic conditions and so on.

In order to test whether or not there are any differences between the mean journey times on the k routes, I chose a route at random for each of N journeys. I recorded the time for each journey, entered the data into a spreadsheet, and carried out an ANOVA analysis. Part of the output was as follows.

Source of variation	Sum of squares	Degrees of freedom
Between groups	333.77	
Within groups		15
Total	752.96	18

(i) State the values of k and N. **[2]**

(ii) Complete the analysis using a 5% significance level. **[10]**

END OF QUESTION PAPER

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A2 GCE MATHEMATICS (MEI)

4769/01 Statistics 4

PRINTED ANSWER BOOK

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OCR supplied materials:
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Other materials required:
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Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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1 (i)	
1 (ii)	
	(answer space continued on next page)

1 (ii)	(contined)
1 (iii)	
	(answer space continued on next page)

1 (iii)	(continued)
1 (iv)	

1 (v)	
2 (i)	
	(answer space continued on next page)

2 (i)	**(continued)**
2 (ii)	

2 (iii)	
2 (iv)	
	(answer space continued on next page)

2 (iv)	**(continued)**

3 (a)(i)	

3 (a)(ii)	
3 (b)(i)	

3 (b)(ii)	

4 (a)(i)	
4 (a)(ii)	

4 (a)(iii)	
4 (b)(i)	

4 (b)(ii)	
	(answer space continued on next page)

(answer space continued on next page)

4 (b)(ii)	(continued)

OCR

Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4769**: Statistics 4

Advanced GCE

Mark Scheme for June 2015

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		The integral is $-\exp(-x^2/a)$, limits 0 and infinity. (Which evaluates to 0 – (–1) =1.)	B1 [1]	Answer given	
	(ii)		$\mathrm{E}(X^2)=\int_0^\infty \frac{2}{a}x^3\exp\left(-\frac{x^2}{a}\right)\mathrm{d}x$ *or* $\int_0^\infty x^2\frac{2}{a}x\exp\left(-\frac{x^2}{a}\right)\mathrm{d}x$	M1		
			$=\left[x^2\left(-\exp\left(-\frac{x^2}{a}\right)\right)\right]_0^\infty+\int_0^\infty 2x\exp\left(-\frac{x^2}{a}\right)\mathrm{d}x$	M1A1	M1 parts	
			$= 0 + a = a$	A1	Cao	
			$\mathrm{E}(X^4)=\int_0^\infty \frac{2}{a}x^5\exp\left(-\frac{x^2}{a}\right)\mathrm{d}x$ *or* $\int_0^\infty x^4\frac{2}{a}x\exp\left(-\frac{x^2}{a}\right)\mathrm{d}x$	M1		
			$=\left[x^4\left(-\exp\left(-\frac{x^2}{a}\right)\right)\right]_0^\infty+\int_0^\infty 4x^3\exp\left(-\frac{x^2}{a}\right)\mathrm{d}x$	A1		
			$= 0 + 2a \times a = 2a^2$	A1 [7]	Answer given so working must be convincing	
	(iii)		Likelihood is $\left(\frac{2}{a}\right)^n\prod X_i\exp\left(-\frac{1}{a}\sum X_i^2\right)$	M1A1A1	M1 for recognisable attempt to obtain the likelihood. A1 for $\prod X_i$. A1 for the exp fn.	
			(log likelihood as given) Differentiate log likelihood	M1		
			to obtain $-\frac{n}{a}+\frac{1}{a^2}\sum X_i^2$	A1		
			Set this to zero to obtain $\mathrm{MLE}(a)=\frac{1}{n}\sum X_i^2$	M1A1 [7]	Justification of a maximum not required	

Question			Answer	Marks	Guidance	
	(iv)		E(MLE(a)) = $\frac{1}{n}\sum \mathrm{E}\left(X_i^2\right)=\frac{1}{n}n\,a=a$	B1		
			That is, the MLE is unbiased	E1	Seen or very clearly implied	
			Var(MLE(a)) = $\frac{1}{n^2}\sum \mathrm{Var}\left(X_i^2\right)=\frac{1}{n^2}\sum\left(\mathrm{E}\left(X_i^4\right)-\left(\mathrm{E}\left(X_i^2\right)\right)^2\right)$	M1M1		
			$=\frac{1}{n^2}n\left(2a^2-a^2\right)=\frac{a^2}{n}$	A1		
				[5]		
	(v)		Maximum likelihood estimate (also unbiased) of a is 1.471	B1	Explanation not required	
			with estimated standard error (a / √ n) 0.1471.	B1	if calculations are correct	
			95% CI is 1.471 ± 1.960 × 0.1471 = 1.471 ± 0.288	M1A1	Accept 2 for 1.960	
				[4]		
				[24]		

Question			Answer	Marks	Guidance	
2	(i)		$\mathrm{M}_Y(\theta) = \mathrm{E}(\mathrm{e}^{\theta Y}) = \int_0^\infty \mathrm{e}^{\theta y} \frac{1}{\sqrt{2\pi y}} \mathrm{e}^{-\frac{1}{2}y} \mathrm{d}y$	M1, A1		
			$= \int_0^\infty \frac{1}{\sqrt{2\pi y}} \mathrm{e}^{-\frac{1}{2}y(1-2\theta)} \mathrm{d}y$	A1		
			Substitute $u = y(1 - 2\theta)$, $\mathrm{d}u = \mathrm{d}y(1 - 2\theta)$ to obtain	M1		
			$\int_0^\infty \frac{\sqrt{1-2\theta}}{\sqrt{2\pi u}} \mathrm{e}^{-\frac{1}{2}u} \frac{1}{1-2\theta} \mathrm{d}u$	A1		
			$= (1-2\theta)^{-1/2} \int_0^\infty \frac{1}{\sqrt{2\pi u}} \mathrm{e}^{-\frac{1}{2}u} \mathrm{d}u$	B1		
			$= (1-2\theta)^{-1/2}$		Answer given	
				[6]		
	(ii)		Either expand the mgf as a power series:			
			$(1-2\theta)^{-1/2} = 1 + \theta + 3\frac{\theta^2}{2!} + \ldots$	M1A1		
			$\mathrm{E}(Y) = 1$ (coefficient of θ)	B1		
			$\mathrm{E}(Y^2) = 3$ (coefficient of $\theta^2/2!$)	B1		
			Hence $\mathrm{Var}(Y) = 2$	B1		
			– –	**[5]**		
			Or differentiate the mgf:	M1		
			1st derivative simplifies to $(1-2\theta)^{-3/2}$	A1		
			Putting $\theta = 0$ gives $\mathrm{E}(Y) = 1$	B1		
			2nd derivative simplifies to $3(1-2\theta)^{-5/2}$	A1		
			Putting $\theta = 0$ gives $\mathrm{E}(Y^2) = 3$, hence $\mathrm{Var}(Y) = 2$	B1		
				[5]		
	(iii)		For independent rvs X and Y,			
			$\mathrm{M}_{X+Y}(\theta) = \mathrm{M}_X(\theta)\,\mathrm{M}_Y(\theta)$	B1		

Question			Answer	Marks	Guidance	
			Hence $M_U(\theta) = (1-2\theta)^{-n/2}$	B1		
			$E(U) = n$	B1		
			$Var(U) = 2n$	B1		
				[4]		
	(iv)		$M_W(\theta) = E\left(\exp\left(\theta\left(\frac{U-n}{\sqrt{2n}}\right)\right)\right)$	B1		
			$= \exp\left(-\frac{\theta n}{\sqrt{2n}}\right) E\left(\exp\left(\frac{\theta}{\sqrt{2n}} U\right)\right)$	B1	Or by use of general linear transformation result	
			$= \exp\left(-\frac{\theta n}{\sqrt{2n}}\right) M_U\left(\frac{\theta}{\sqrt{2n}}\right)$	B1		
			$= \exp\left(-\frac{\theta n}{\sqrt{2n}}\right)\left(1-\frac{2\theta}{\sqrt{2n}}\right)^{-n/2}$	B1		
			Hence $\ln(M_W(\theta))$ as given Expanding $\ln(M_W(\theta))$ gives			
			$-\sqrt{\frac{n}{2}}\,\theta + \frac{n}{2}\left(\sqrt{\frac{2}{n}}\,\theta + \frac{1}{2}\left(\sqrt{\frac{2}{n}}\,\theta\right)^2 + \text{terms of order } n^{-3/2}\right)$	M1A1	Award a maximum of 3 marks from here on if no account is taken of terms beyond θ^2.	
			Convincing simplification to $\frac{1}{2}\theta^2$ + terms of order $n^{-1/2}$	B1	(answer given)	
			Hence tends to $\frac{1}{2}\theta^2$.			
			The mgf therefore tends to $\exp(\frac{1}{2}\theta^2)$.	B1	Uniqueness of mgfs may be implied	
			That is, the distribution of W tends to the standard Normal.	B1		
				[9]		
				[24]		

Question			Answer	Marks	Guidance	
3	**(a)**	(i)	Sample means:			
			standard 107.2766, GM 113.6192	B1		
			Sample variances:			
			standard 138.2243, GM 121.9372	B1		
			H_0: $\mu_1 = \mu_2$ (the means of the underlying distributions) H_0: $\mu_1 \neq \mu_2$	B1	Must be clear that the hypotheses refer to underlying means	
			Test statistic: $\dfrac{113.6192 - 107.2766}{\sqrt{\frac{138.2243}{30} + \frac{121.93721}{26}}}$ = 2.08(0103)	M1A1		
			Compare with z distribution	B1		
			Critical value, 2 tails, 5%, 1.960	B1		
			So 2.08(0103) is just in the critical region			
			Hence reject H_0 and conclude that there is a difference in mean weight between standard and GM tomatoes.	B1B1		
			Assumption:			
			the tomatoes may be regarded, in some sense, as random samples of their respective varieties.	B1 **[10]**		
		(ii)	Additional assumptions:			
			underlying Normality of the tomatoes' weights	B1		
			common variance in the underlying distributions	B1		
			Given the sample sizes it seems safe to use the Normal distribution.	E1		
			The Normal test is better in that it makes fewer assumptions.	E1 **[4]**		
	(b)	(i)	The tomatoes should be chosen at random.	B1		
			The panel should not know which tomatoes are GM / standard.	B1		
			Making fine judgements on the appearances of tomatoes is unlikely to be reliable.	B1 **[3]**	Accept other sensible comments	

Question			Answer	Marks	Guidance	
		(ii)	Rank sums: Standard 134, GM 76	M1A1		Or Mann-Whitney 79, 21
			H_0: GM and std tomatoes have, on the whole, the same appearance H_1: GM tomatoes have, on the whole, a better appearance than std tomatoes	B1	Hope for, but don't expect, a formulation in terms of a shift in location parameter for underlying distributions of appearances	
			(lower rank sum for GM indicates that the evidence is in the correct tail)	B1		
			Wilcoxon rank sum test	B1	May be implied	
			Critical value for $m = n = 10$, 1 tail, 1% level is 74	B1		Critical value 19
			The observed value of 76 is not in the critical region so accept H_0. That is, conclude that there is insufficient evidence to suppose that, on the whole, GM tomatoes have a better appearance than standard tomatoes.	B1		
				[7] **[24]**		

Question			Answer	Marks	Guidance	
4	(a)	(i)	Y_{ij} denotes the jth observation (or measurement) in the ith group	B1	Accept just Y_{ij} denotes the observations (measurements)	
			μ is the underlying mean for whole population	B1	Withhold these marks if there is no reference to populations	In marking section (a), evidence of understanding is more important than precise wording.
			α_i denotes the population mean difference for the ith group	B1		
			ε_{ij} denotes the random error in the ijth observation (or measurement)	B1		
				[4]		
		(ii)	ε_{ij} ~ ind $N(0, \sigma^2)$,	B1B1	B1 ind N, B1 zero mean	
			where σ^2 is constant across groups.	B1	May be implied by later comment	
			The variance ratio test (F test) requires underlying Normality (and common variance).	B1	Common variance may be implied by earlier statement	
				[4]		
		(iii)	H_0: all the α_i are zero	B1	Accept clear wording in terms of all group effects being zero, or all groups having same underlying distribution	
			H_1: not all the α_i are zero	B1		H_1: all the α_i are non-zero scores zero
			If H_0 is accepted then we proceed on the basis that the group means are all equal. I.e. there are no ‘treatment effects’.	E1		
			If H_0 is rejected the we proceed on the basis that the group means are not all equal. I.e. there are some ‘treatment effects’.	E1		
				[4]		
	(b)	(i)	k = 4 (number of routes is 4)	B1		
			N = 19 (number of journeys is 19)	B1		
				[2]		

Question			Answer	Marks	Guidance	
		(ii)		Source of var'n	SS	df	MS	F
---	---	---	---	---
Between groups	333.77	**3**	**111.2567**	**3.98(113)**
Within groups	**419.19**	15	**27.9460**	
Total	752.96	18				M1 A4	Evidence of understanding how to complete the table for the figures shown in bold -1 each error	
			H_0: all routes have the same underlying mean time H_1: routes do not all have the same underlying mean time The observed variance ratio is compared with $F_{3,\ 15}$ The 5% critical value is 3.29 (tables) or 3.287382 (calculator) Observed value is in critical region, so reject H_0. That is, proceed on the assumption that the routes do differ in underlying mean time.	B1 M1 A1 B1 B1 **[10]** **[24]**	Hypotheses may be implied by later comments May be implied by use of a correct value Or equivalent wording	

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GCE Mathematics (MEI)											
				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	48	43	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	50	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	56	51	46	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	74	67	60	54	48	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	62	57	53	49	45	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	38	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	62	54	46	39	32	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	54	47	40	33	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	65	60	55	50	46	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	64	58	52	47	42	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	56	51	46	41	37	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	54	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	45	40	34	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

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GCE Statistics (MEI)											
				Max Mark	a	b	c	d	e	u
G241	01	Statistics 1 MEI (Z1)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
G242	01	Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
G243	01	Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
			UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)										
				Max Mark	a	b	c	d	e	u
G244	01	Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02	Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
G245	01	Statistics 1 MEI	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
G246	01	Decision 1 MEI	Raw	72	56	51	46	41	37	0
			UMS	100	80	70	60	50	40	0